Exponential function: $f(x) = 2^x$ Logarithmic function: $g(x) = \log_2 x$

These two functions are **inverse functions**. Let's look at the characteristics of inverse functions to see how these functions fulfill some requirements to be **inverse functions**.

- All the $(x, y)$ ordered pairs of one function work as $(y, x)$ ordered pairs in the other function.

$$f(x) = 2^x$$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ [or $f(x)$] | 2 | 4 | 8 | 16 | 32 |

$$g(x) = \log_2 x$$

| $x$ | 2 | 4 | 8 | 16 | 32 |
|---|---|---|---|---|---|
| $y$ [or $f(x)$] | 1 | 2 | 3 | 4 | 5 |

Let’s use algebra to prove this happens for all $(x, y)$ combinations when we invert them to $(y, x)$:

$$g(x) = \log_2 x$$
$$y = \log_2 x$$

Invert $x$ and $y$.

$$x = \log_2 y$$

Solve for $y$.

$$x = \log_2 \mathbf{y}$$

can be rewritten as

Exponent

$$\log_2 \mathbf{y} = x$$

Base

What Base$^{\text{Exponent}}$ is

We learned on the first day of studying logarithms that this can be rearranged as

Exponent

$$2^x = \mathbf{y}$$

Base

What Base$^{\text{Exponent}}$ is

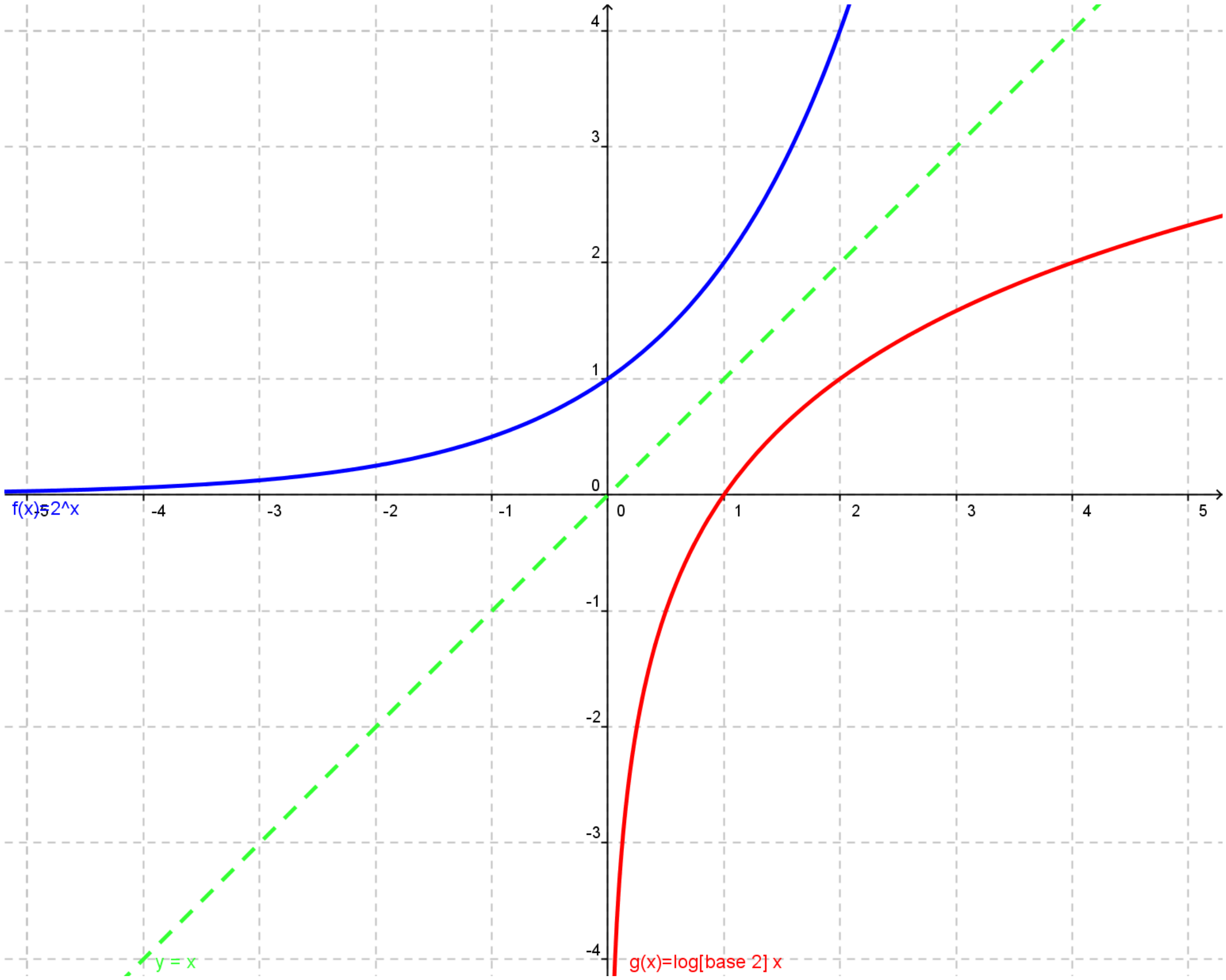
That can be rewritten as

$$\mathbf{y} = \mathbf{2}^{\boldsymbol{x}}$$

That is the same thing as

$$f(x) = 2^x$$

- The graphs of the two functions should be symmetrical to the line $x = y$.




Example 1 – Simplify $11^{\log_{11} x}$

This may look bad, but we can do this in our heads once we realize what's happening!

1) We know that the exponential function $f(x) = 11^x$ and the logarithmic function $g(x) = \log_{11} x$ are **inverse functions**.

2) It looks like $g(x)$ is the input for the $f(x)$ function.

$$f(x) = 11^x$$
$$f(g(x)) = 11^{\log_{11} x}$$

3) We know that when $f(x)$ and $g(x)$ are inverse functions, then

$$f(g(x)) = \boldsymbol{x} \quad \text{and} \quad g(f(x)) = \boldsymbol{x}$$

4) The answer must just be $\boldsymbol{x}$ !

$$\boxed{\boldsymbol{x}}$$

Example 2 – Simplify $\log_{12} 12^x$

This may look as bad as Example 1, but it is just as easy!

1) We know that the exponential function $f(x) = 12^x$ and the logarithmic function $g(x) = \log_{12} x$ are **inverse functions**.

2) It looks like $f(x)$ is the input for the $g(x)$ function.

$$g(x) = \log_{12} x$$
$$g(f(x)) = \log_{12} 12^x$$

3) We know that when $f(x)$ and $g(x)$ are inverse functions, then

$$f(g(x)) = \boldsymbol{x} \quad and \quad g(f(x)) = \boldsymbol{x}$$

4) The answer must just be $\boldsymbol{x}$ !

$$\boxed{\boldsymbol{x}}$$